January 1993

Mathematics 30

Grade 12 Diploma Examination

Alberta
EDUCATION

January 1993

Mathematics 30

Grade 12 Diploma Examination

Description

Time allotted: 2.5 h
Total possible marks: 70

This is a **closed-book** examination consisting of **three** parts:

Part A
has 42 multiple-choice questions each with a value of one mark.

Part B
has 7 numerical-response questions each with a value of one mark.

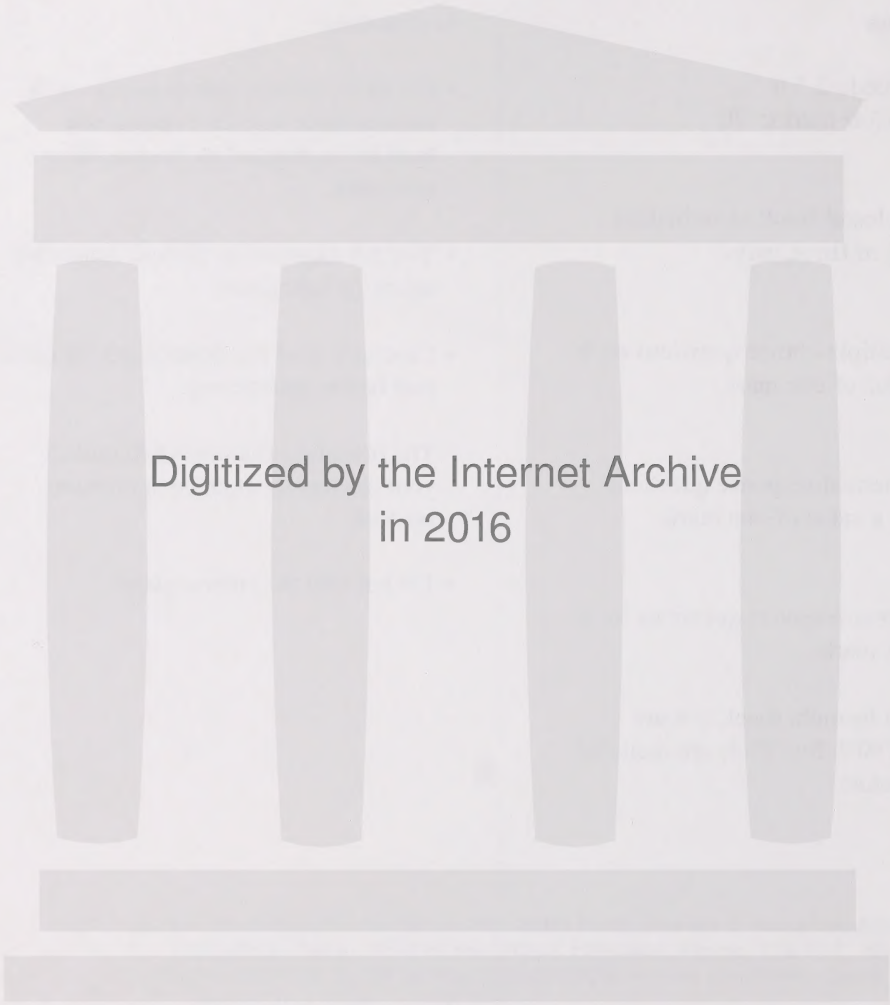
Part C
has 4 written-response questions for a total of 21 marks.

A tear-out formula sheet, z-score page, and 90% Box Plots are included in this booklet.

Instructions

- Fill in the information required on the answer sheet and the examination booklet as directed by the presiding examiner.
- You are expected to provide your own scientific calculator.
- Carefully read the instructions for each part before proceeding.
- The presiding examiner will collect your answer sheet and examination booklet.
- Do not fold the answer sheet.

Note: *The perforated pages at the back of this booklet may be torn out and used for your rough work.* ***No marks*** *will be given for work done on the tear-out pages.*



Part A: Multiple Choice

42 Questions

Instructions

- Consider all numbers used in the questions to be **exact real** numbers and not the result of a measurement.
- Read each question carefully and decide which of the choices **best** completes the statement or answers the question.
- Locate that question number on the separate answer sheet provided and fill in the circle that corresponds to your choice.

  Example

  This diploma examination is for the subject of

  A. biology
  B. physics
  C. chemistry
  D. mathematics

  Answer Sheet

  Ⓐ Ⓑ Ⓒ ●

- **Use an HB pencil only.**
- If you wish to change an answer, erase **all** traces of your first answer.

Note: *The perforated pages at the back of this booklet may be torn out and used for your rough work.* ***No marks*** *will be given for work done on the tear-out pages.*

Do not turn the page to start the examination until told to do so by the presiding examiner.

1. Krista used the graph of a third-degree polynomial function to make the following table of values:

x	–3	–2	–1	0	1	2	3
$P(x)$	–12	0	2	0	0	8	30

The equation for the polynomial function that Krista used is

A. $P(x) = x(x + 1)(x + 2)$

B. $P(x) = x(x - 1)(x - 2)$

C. $P(x) = x(x - 1)(x + 2)$

D. $P(x) = x(x + 1)(x - 2)$

2. When the polynomial $P(x) = 3x^4 - 40$ is divided by $x - 2$, the coefficient of the x^2 term in the quotient is

A. 0

B. 6

C. 8

D. 12

3. Alvin is trying to find the rational zeros of $P(x) = 5x^3 - 6x^2 + 4$. Which of the following rational numbers should he consider?

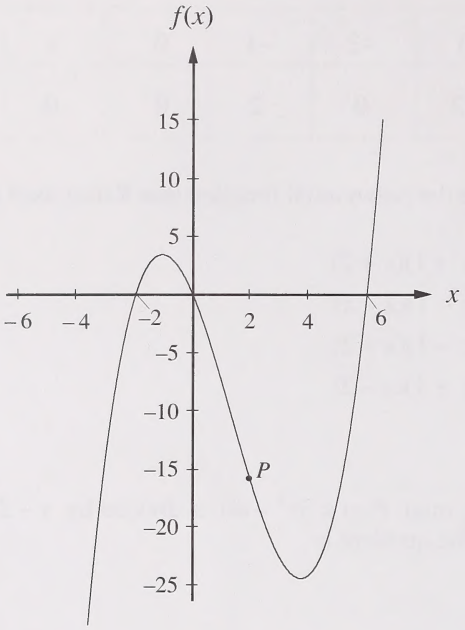
A. $\frac{5}{4}$

B. $\frac{3}{4}$

C. $\frac{2}{3}$

D. $\frac{2}{5}$

4. The polynomial function $f(x) = ax^3 + mx^2 - 6x$ has integral zeros. The graph of $f(x)$ passes through the point $P(2, -16)$ and is shown below:



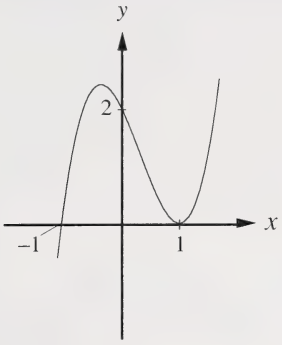
The value of m in $f(x) = ax^3 + mx^2 - 6x$ is

A. 5
B. 2
C. –2
D. –5

5. Two factors of the polynomial $2x^3 - hx + k$ are $(x + 2)$ and $(x - 1)$. The value of h in this polynomial is

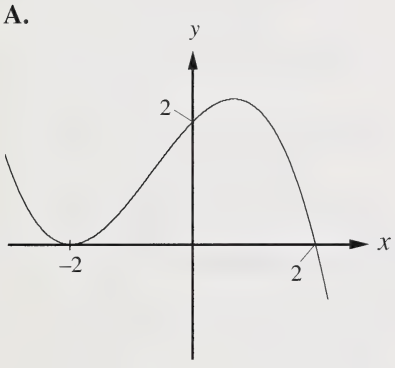
A. 4
B. 6
C. 14
D. 18

6. This is the graph of $y = P(x)$:

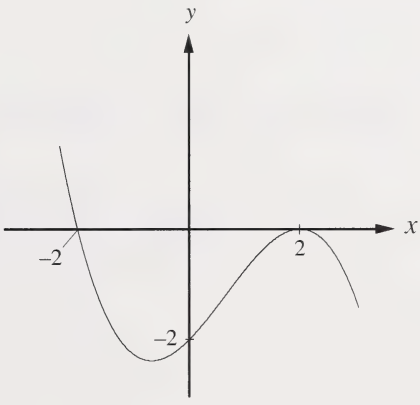


The graph of $y = -2P(x)$ is

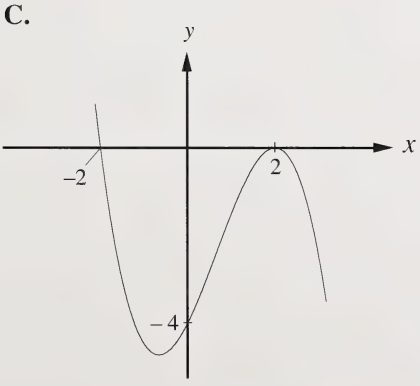
A.



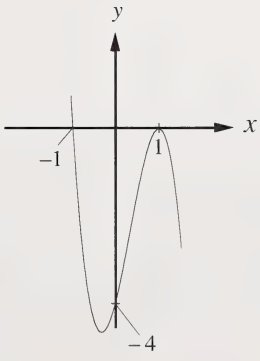
B.



C.



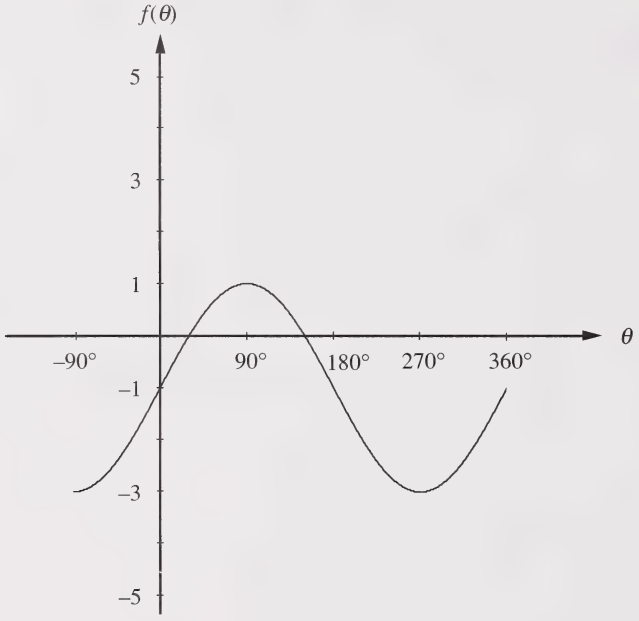
D.



7. When the polynomial $G(x)$ is divided by $(x-1)$, the remainder is 3. If the polynomial $P(x) = 2G(x)$ is divided by $(x-1)$, the remainder is

A. 6
B. 3
C. 2
D. 1

8. The graph of $f(\theta) = 2 \sin \theta - 1$ is shown below.



The range for this function is

A. $-3 < f(\theta) < 1$
B. $-3 \leq f(\theta) \leq 1$
C. $-90° \leq \theta \leq 360°$
D. $-90° < \theta < 360°$

9. The expression $\sqrt{\dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}}$, $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$, is equal to

A. $\csc\theta$

B. $\sec\theta$

C. $1 + \tan\theta$

D. $1 + \sin\theta$

10. Which of the following statements about the graph of $f(\theta) = 3\cos\left(\theta - \dfrac{\pi}{2}\right)$ is **false**?

A. The amplitude of the graph is 3.

B. The period of the graph is 2π.

C. The graph of $f(\theta) = 3\cos\left(\theta - \dfrac{\pi}{2}\right)$ is the graph of $f(\theta) = 3\cos\theta$ with a phase shift of $\dfrac{\pi}{2}$ to the right.

D. The graph of $f(\theta) = 3\cos\left(\theta - \dfrac{\pi}{2}\right)$ is the graph of $f(\theta) = \cos\left(\theta - \dfrac{\pi}{2}\right)$ with a vertical translation of 3.

11. If $2\sin^2\theta + 3\sin\theta = 2$, and θ is acute, then $\cos\theta$ is

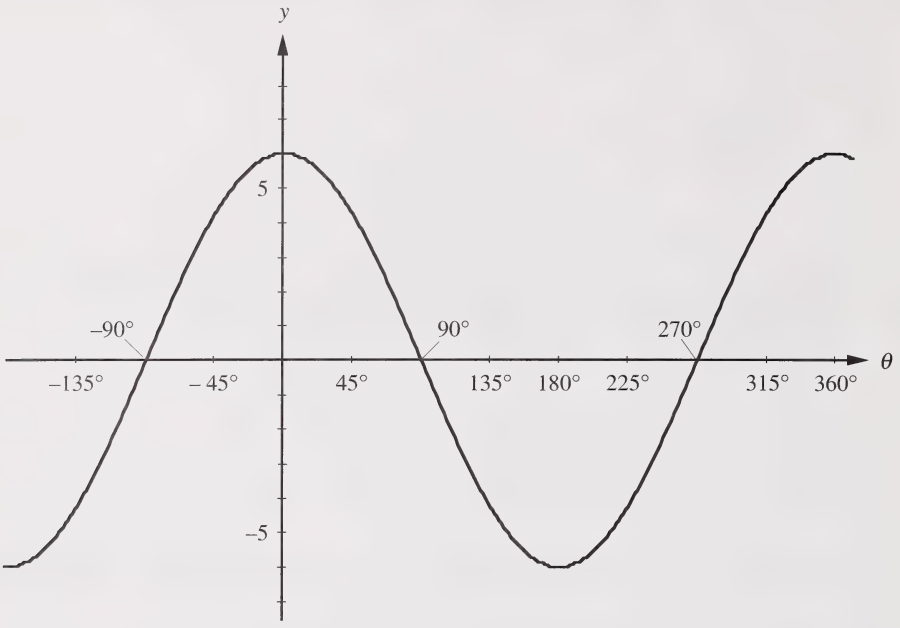
A. $\dfrac{1}{2}$

B. $\sqrt{3}$

C. $\dfrac{\sqrt{3}}{2}$

D. $\dfrac{\sqrt{2}}{2}$

12. The graph of $y = a \cos \theta$ is shown below.



$P(56°, y)$ represents a point on this curve. Correct to the nearest tenth, which of the following could be a value of y?

A. 0.6
B. 2.8
C. 3.4
D. 6.0

13. Thirty high school students recorded the number of compact discs and the number of audio tapes they each owned. One student then placed the data on a scatter plot, with the number of compact discs (d) on the horizontal axis and the number of audio tapes (t) on the vertical axis. Next, she constructed a line of best fit from the data. Two points on the line of best fit were (3, 12) and (8, 37). The equation of this line of best fit is

A. $t = 5d - 3$
B. $t = 5d - 177$
C. $t = -5d + 77$
D. $t = -5d + 177$

14. A sample of 20 randomly chosen voters in the constituency of Top Soil were asked if they would vote for a particular candidate in the next provincial election. Four voters answered yes. Based on this sample, the 90% confidence interval for the percentage of yes responses in the voting population of Top Soil is

A. 10% – 35%

B. 10% – 40%

C. 0% – 45%

D. 5% – 35%

15. A city car dealer hired a student to survey potential new car buyers. The student surveyed 1000 Grade 12 students and reported the results to the car dealer. From the car dealer's perspective, the student's selection for the sample was

A. inappropriate because Grade 12 students are not necessarily representative of the potential new car buyers in this city

B. appropriate because Grade 12 students are representative of the potential new car buyers in this city

C. inappropriate because the student did not survey people of all ages

D. appropriate because the student chose a stratified sample

16. A Mathematics 30 class constructed box plots from a series of experiments. Which experiment would most likely produce a box plot that is **not** symmetric about the median?

A. Tossing a coin 1000 times and recording the number of times that heads appears

B. Spinning a spinner 1000 times on a quarter-blue, quarter-red, quarter-green, and quarter-yellow circle, and recording the number of times the spinner lands on the yellow quarter

C. Making 20 random choices of five marbles from a bag that contains 45 black marbles and 10 red marbles, and recording the number of red marbles

D. Rolling a die 20 times and recording the number of times an even number is rolled

17. The results of a survey on the speeds of cars crossing a highway bridge were normally distributed with a mean speed of 100 km/h and a standard deviation of 10 km/h. If in 1 h, 13 cars crossed the bridge at a minimum speed of 115 km/h, then the best estimate of the total number of cars that crossed the bridge in that hour is

A. 195

B. 121

C. 56

D. 30

18. To the nearest tenth, the percentage of scores in a normal distribution between $\mu - 0.91\sigma$ and $\mu + 0.76\sigma$ is

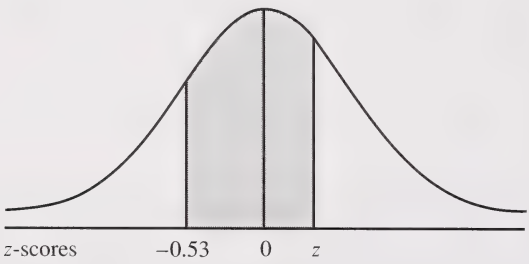
A. 15.0%

B. 56.8%

C. 59.5%

D. 68.0%

19. The shaded area in the standard normal distribution shown below is 0.2967 of the total area beneath the normal curve.



The value of z is

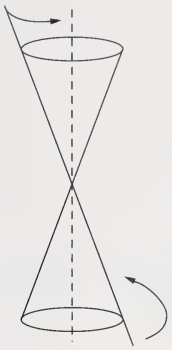
A. 0.8300

B. 0.2400

C. 0.2019

D. 0.0948

20. A circular conical surface is shown below.



A plane is parallel to the generator of this circular conical surface. The nondegenerate conic section formed by the intersection of the plane and the conical surface is

A. a circle

B. a parabola

C. an ellipse

D. a hyperbola

Use the following information to answer question 21.

Khalid used a graphing calculator to investigate the numerical coefficients in the equation $Ax^2 + Cy^2 + F = 0$. Khalid then made the following observations about nondegenerate quadratic relations:

I. If $A = C$ and $F < 0$, a circle may be formed.
II. If $AC < 0$ and $F < 0$, a hyperbola may be formed.
III. If $A = 0$ and $FC > 0$, a parabola may be formed.
IV. If $A > 0, C > 0$, and $F < 0$, an ellipse may be formed.

21. Which of Khalid's observations contains an error?

A. I

B. II

C. III

D. IV

22. A hyperbola degenerates into

A. one line
B. one point
C. two parallel lines
D. two intersecting lines

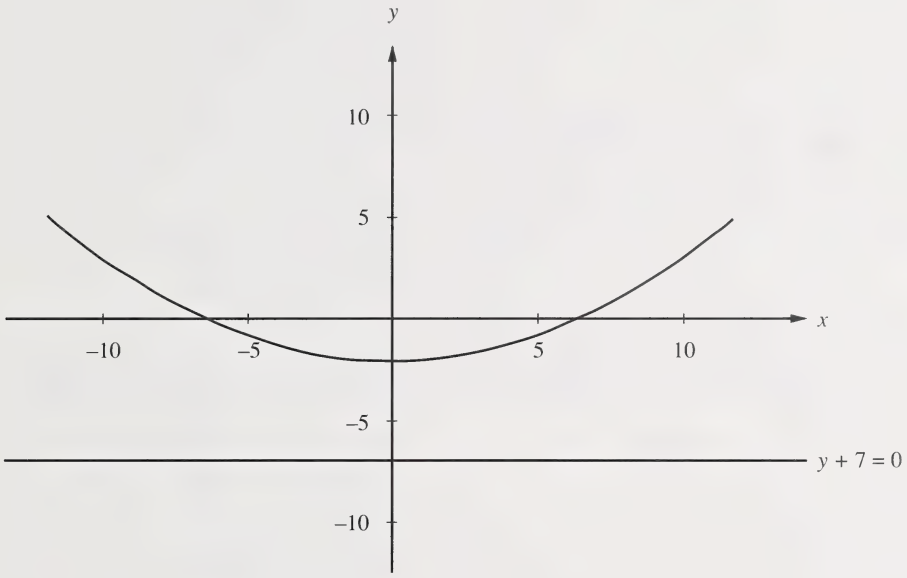
23. A quadratic relation is defined by the equation $Ax^2 + Cy^2 + Ey + F = 0,\ F < 0$. The graph of a quadratic **function** of x may exist if the numerical coefficients satisfy

A. $A = 1,\ AC > 0$
B. $E = 1,\ AC < 0$
C. $A = C = E = 1$
D. $A = E = 1,\ C = 0$

24. While investigating a conic section, Walter determines that the distance from a point on the conic to the focus is 7 units and that the distance from the same point to the directrix is 8 units. The conic section Walter is investigating is

A. a circle
B. an ellipse
C. a parabola
D. a hyperbola

25. In the graph of the parabola shown below, the directrix is at $y + 7 = 0$ and the focus is at (0, 3).



If a point on the graph is at $(m, -1)$, then an exact value of m is

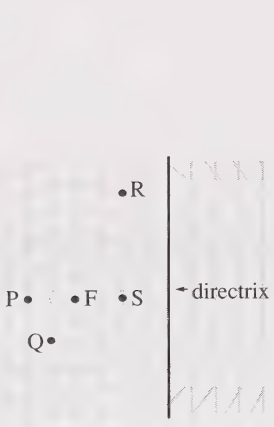
A. $-2\sqrt{5}$

B. $-5\sqrt{2}$

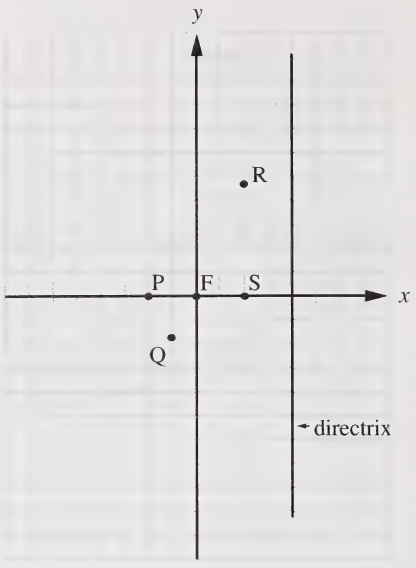
C. $-3\sqrt{2}$

D. $-3\sqrt{3}$

Use either of the following graphs to answer question 26.



Graph 1



Graph 2

26. The focus, F, and the directrix of a conic are shown. If the eccentricity is equal to $\frac{2}{5}$, then a point on the graph of the conic would be

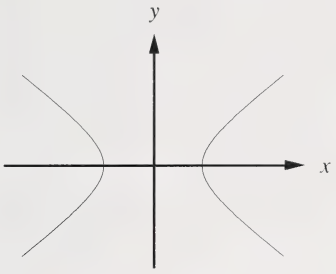
A. Q

B. R

C. S

D. P

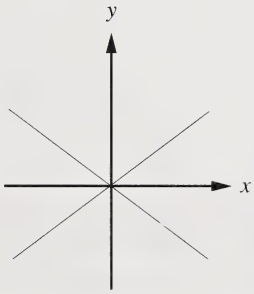
27. A student graphed a hyperbola using a computer. This is the computer display of that hyperbola and its equation:



$36x^2 - 64y^2 - 2304 = 0$

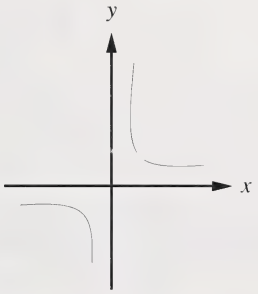
Hyperbola 1

The graphs of three other hyperbolas and their equations are shown below.



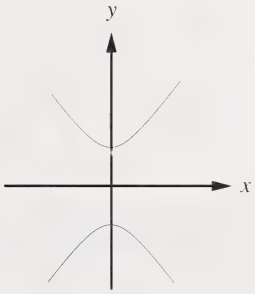
$9x^2 - 16y^2 = 0$

Hyperbola 2



$14x^2 - 100xy + 14y^2 + 2304 = 0$

Hyperbola 3



$64x^2 - 36y^2 + 2304 = 0$

Hyperbola 4

If the student uses the computer to rotate hyperbola 1 about the origin, then the student will likely obtain

A. hyperbola 2 only

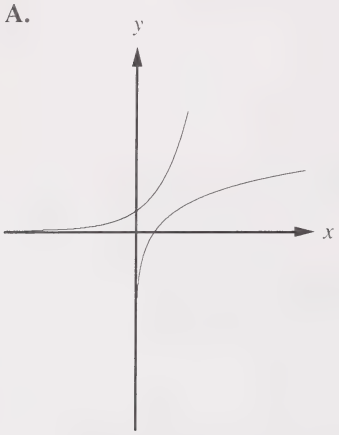
B. hyperbola 3 only

C. hyperbolas 2 and 4

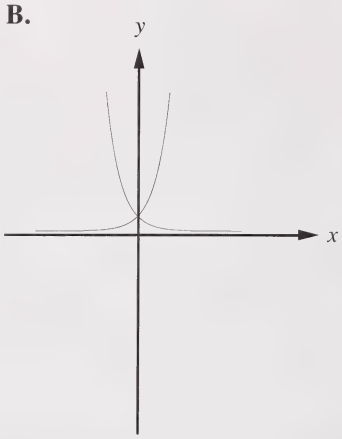
D. hyperbolas 3 and 4

28. The graphs of an exponential function and its inverse are shown in

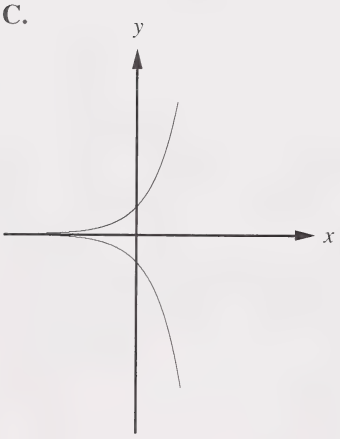
A.



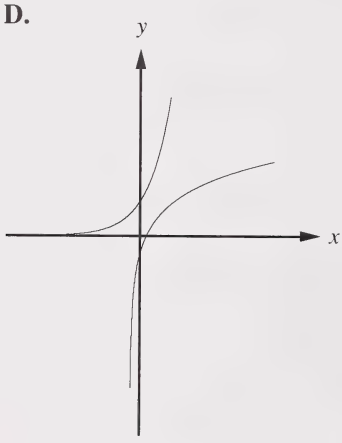
B.



C.



D.



29. In $\log_3(3x + 6) - \log_3(x - 4) = 2$, the value of x is

A. 0
B. 4
C. 7
D. 10

30. The graph of the exponential function $f(x) = (b)^{x-2},\ b > 0,\ b \neq 1$ intersects the y-axis at

A. b^2

B. $\frac{1}{b^2}$

C. $-\frac{1}{b^2}$

D. $-b^2$

31. The number of different arrangements of all the letters in the word **ATLANTA** is

A. 12

B. 112

C. 420

D. 5040

32. Ten children are standing in a circle waiting to play a game. The supervisor picks 3 of these children to stand in the middle of the circle. In how many different ways could the supervisor choose the 3 children?

A. ${}_{10}C_3$

B. ${}_{10}P_3$

C. ${}_{9}C_3$

D. ${}_{9}P_3$

33. If a regular polygon has 44 diagonals, then the number of sides in the polygon is

A. 8

B. 9

C. 10

D. 11

34. You are one of a group of 15 people, 3 of whom will be randomly selected to form a committee. The probability, correct to the nearest ten thousandth, that you will be one of the 3 people selected is

A. 0.2000

B. 0.0667

C. 0.0022

D. 0.0004

35. The number of terms in the expansion of $\sum_{n=4}^{12}(4n)$ is

A. 8

B. 9

C. 12

D. 16

36. Kelly decides to ride her stationary bicycle every day. She begins by riding for 5 min on the first day. Each day after that, she increases the amount of time she rides by 2 min. At the end of 30 days, the **total time** Kelly has spent riding her bike is

A. 1 h 3 min

B. 6 h

C. 8 h 45 min

D. 17 h

37. If 51, e, f, g, h, 291 form an arithmetic sequence, then the value of f is

A. 111

B. 131

C. 137

D. 147

38. In an arithmetic series, the first term is 84 and the second term is 78. If the sum of the first n terms is 612, then an equation that could be used to determine the value(s) of n is

A. $n^2 + 27n - 204 = 0$

B. $n^2 - 27n + 180 = 0$

C. $n^2 - 28n + 192 = 0$

D. $n^2 - 29n + 204 = 0$

39. In a geometric sequence, the thirteenth term is 243 and the common ratio is $\sqrt{3}$. The value of the first term is

A. $\frac{1}{3}$

B. 3

C. 13

D. $222\frac{2}{9}$

40. If the third term in a geometric sequence is 288 and the seventh term is 1458, then the fifth term is

A. 432

B. 648

C. 873

D. 972

41. The sum of the first 25 terms of the geometric series $\frac{4}{5} + \frac{12}{5} + \frac{36}{5} + \ldots$ can be expressed as

A. $\frac{2}{5}(3^{24})$

B. $\frac{8}{5}(3^{24})$

C. $\frac{2}{5}(3^{25} - 1)$

D. $\frac{8}{5}(3^{25} - 1)$

(Question 42 is on page 18.)

42. A sequence is defined by $t_1 = -5,\ t_n = 3 - t_{n-1};\ n \geq 2,$ where n is a natural number. The sixteenth term, t_{16}, of this sequence is

A. –5
B. –2
C. 1
D. 8

You have now completed Part A. Proceed directly to Part B.

Part B: Numerical Response

7 Questions

Instructions

- Consider all numbers used in the questions to be **exact real** numbers and not the result of a measurement.
- Read each question carefully.
- Record your answer on the answer sheet provided by writing it in the boxes and then filling in the corresponding circles.
- **Enter the first digit of your answer in the left-hand box and leave any unused boxes blank.**
- Use an HB pencil only.
- If you wish to change an answer, erase **all** traces of your first answer.

Sample Questions and Solutions

If θ is acute and $\sin\theta = 0.6735$, then the measure of θ correct to the nearest tenth of a degree is _____.

$\theta = 42.33777464 \ldots °$

Record 42.3 on the answer sheet →

For the arithmetic series $-8 + (-5) + (-2) + \ldots + (85)$, the number of terms is _____.

$85 = -8 + (n - 1)(3)$

$93 = 3n - 3$

$n = 32$

Record 32 on the answer sheet →

Start Part B immediately.

1. For the polynomial function $P(x) = (x - 2)^n(x + 3)$, $P(0) = -96$. The value of n is _____.

2. Correct to the nearest tenth of a radian, 130° is equal to _____ rad.

ECORD YOUR ANSWER ON THE ANSWER SHEET

3. A poll using 1030 people was conducted in Fort McMurray . The results were 56% in favor of an expanded transit system. A poll of this size is considered accurate to within 3.1 percentage points, 19 times out of 20. This poll has a $k\%$ confidence level. The value of k, correct to the nearest whole number, is _____.

4. A point $P(3, 4)$ on the graph of a parabola is 10.6 units from the focus. The distance from point P to the directrix of this parabola, correct to the nearest tenth of a unit, is _____.

5. If 6^7 is expressed as a power of 2, then the exponent, correct to the nearest tenth, is _____.

6. The number of different positive 3-digit integers that can be formed that do **not** contain the number 5 is _____.

(Question 7 is on page 22.)

7. If $S_n = n(n + 3)$ represents the sum of a series, then the value of the tenth term, t_{10}, correct to the nearest tenth, is _____.

You have now completed Part B. Proceed directly to Part C.

Part C: Written Response

4 Questions

Instructions

- Consider all numbers used in the questions to be **exact real** numbers and not the result of a measurement.
- Read each question carefully.
- Write your answers in the examination booklet as neatly as possible.
- For full marks, your answers **must show all** pertinent explanations, calculations, and formulas.
- Your answers **should be** presented in a well-organized and appropriate manner.

Note: *The perforated pages at the back of this booklet may be torn out and used for your rough work.* ***No marks*** *will be given for work done on the tear-out pages.*

Start Part C immediately.

For Department Use Only

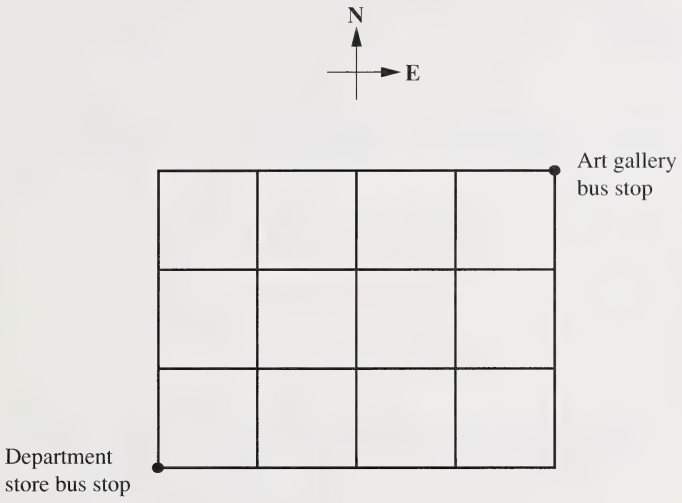
5 marks

1. The current population of the town of Winterlog is 3600. According to studies conducted over the last few years, Winterlog's population is growing at the rate of 1.6% per annum. Assuming that the population will continue to grow at this rate, determine how many **complete** years it will take Winterlog to reach a population of 4800.

For Department Use Only

4 marks

2. The head of city planning and development is designing a bus route from a department store to an art gallery. The art gallery bus stop will be three blocks north and four blocks east of the department store bus stop. The bus will travel only north or east.



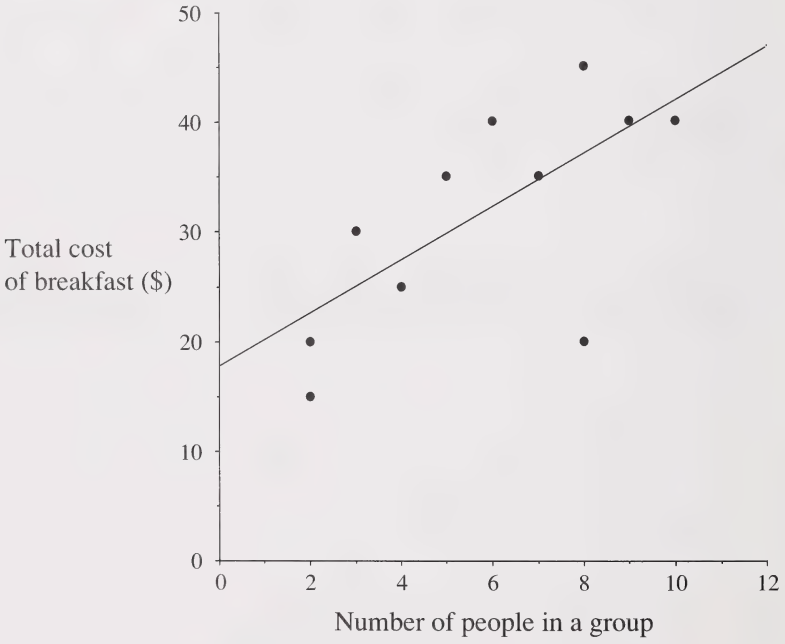
Determine the total number of possible routes from the department store bus stop to the art gallery bus stop. Show how you solved this problem.

For Department Use Only

Total: 5 marks

3. Chewy Bits Cafe offers a variety of breakfast specials. Groups of people come to the cafe for these specials. The chart shows the number of people in 11 groups and the total cost of their breakfasts. The scatter plot and line of best fit are also shown.

Number of people in a group	2	2	3	4	5	6	7	8	8	9	10
Total cost of breakfast ($)	15	20	30	25	35	40	35	45	20	40	40



For Department Use Only

a. Use the line of best fit to predict the total cost of breakfast for a group of 8 people. **(1 mark)**

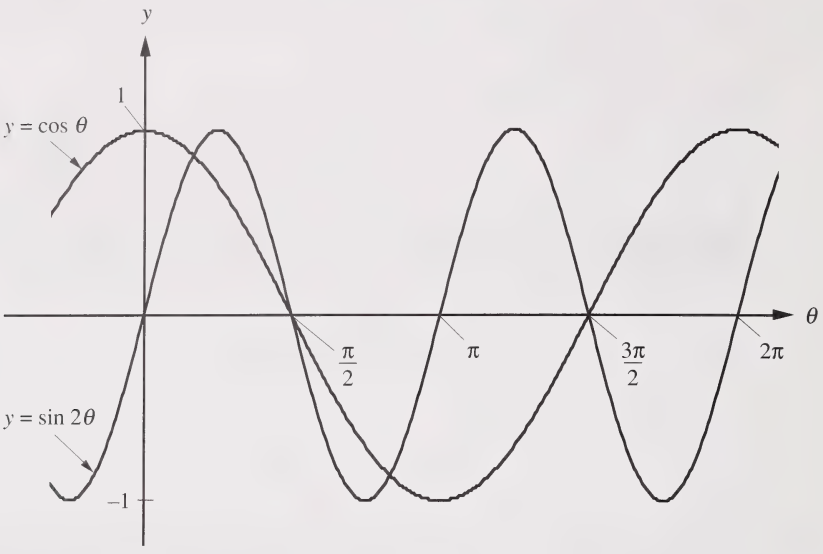
b. Use the line of best fit to determine how many people could eat breakfast for a total cost of $30. **(1 mark)**

c. The line of best fit can be used to make predictions. Use the line of best fit to explain why the actual cost of a breakfast can differ significantly from the predicted cost. **(3 marks)**

For Department Use Only

Total: 7 marks

4. Mike's assignment is to find the **points** of intersection of the graphs of $y = \sin 2\theta$ and $y = \cos \theta$, if $0 \leq \theta < 2\pi$. He uses his computer to graph $y = \sin 2\theta$ and $y = \cos \theta$. His computer display is



(5 marks)

a. Looking at this computer display, Mike notices that one of the points of intersection is $\left(\frac{\pi}{2}, 0\right)$. He knows that it is impossible to find the exact values of all the points of intersection where $0 \leq \theta < 2\pi$ using the graphs displayed, so he decides to find the points of intersection algebraically. He also knows the trigonometric identity $\sin 2\theta = 2 \sin \theta \cos \theta$. Mike's first step in finding the points of intersection of the graphs is as follows:

> Since $y = \sin 2\theta$ and $y = \cos \theta$, the θ values of the points of intersection are found when $\sin 2\theta = \cos \theta$.

Complete Mike's solution.

Use the top of page 29 to complete Mike's solution.

For Department Use Only

b. How are the points of intersection of the graphs $y = 3 \sin 2\theta$ and $y = 3 \cos \theta$ related to the points of intersection of the graphs $y = \sin 2\theta$ and $y = \cos \theta$? **(2 marks)**

You have now completed the examination.
If you have time, you may wish to check your answers.

Mathematics 30 Formula Sheet

The following information may be useful in writing this examination.

I. Polynomial Functions

- $P(x) = D(x)Q(x) + R$
- The roots of a quadratic equation are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

II. Trigonometry

- arc length $a = r\theta$
- $\sec A = \dfrac{1}{\cos A}$
- $\csc A = \dfrac{1}{\sin A}$
- $\cot A = \dfrac{\cos A}{\sin A}$
- $\sin^2 A + \cos^2 A = 1$
- $1 + \tan^2 A = \sec^2 A$
- $1 + \cot^2 A = \csc^2 A$
- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$

III. Statistics

- $z = \dfrac{x - \mu}{\sigma}$
- $y = mx + b$

IV. Quadratic Relations

- $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
- $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- eccentricity $e = \dfrac{|\overline{PF}|}{|\overline{PD}|}$, where F = focus,
  D = directrix, and
  P = point on the conic

V. Permutations and Combinations

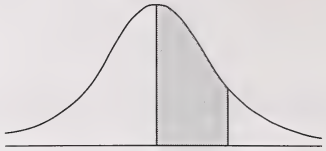
- $n! = n(n-1)(n-2)\ldots(3)(2)(1)$
- ${}_nP_r = \dfrac{n!}{(n-r)!}$
- ${}_nC_r = \dfrac{n!}{r!(n-r)!}$
- $(x + y)^n = {}_nC_0x^n + {}_nC_1x^{n-1}y + {}_nC_2x^{n-2}y^2 + \ldots + {}_nC_kx^{n-k}y^k + \ldots + {}_nC_ny^n$

  General Term

  $t_{k+1} = {}_nC_kx^{n-k}y^k$

VI. Sequences and Series

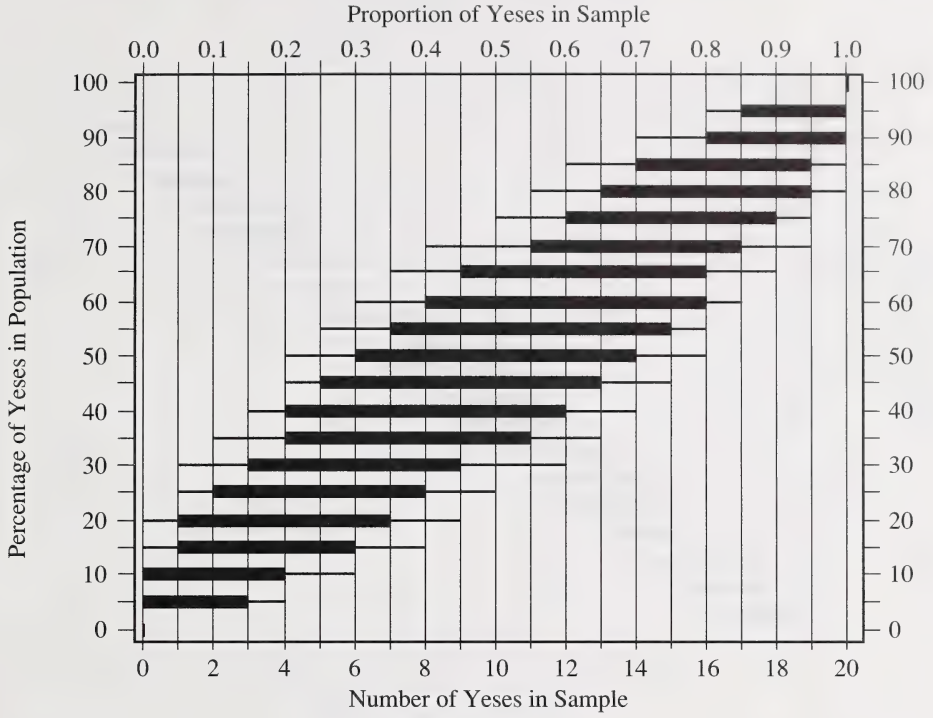
- $t_n = a + (n-1)d$
- $S_n = \dfrac{n(a + t_n)}{2}$
- $S_n = \dfrac{n[2a + (n-1)d]}{2}$
- $t_n = ar^{n-1}$
- $S_n = \dfrac{a(r^n - 1)}{r - 1}$, $r \neq 1$



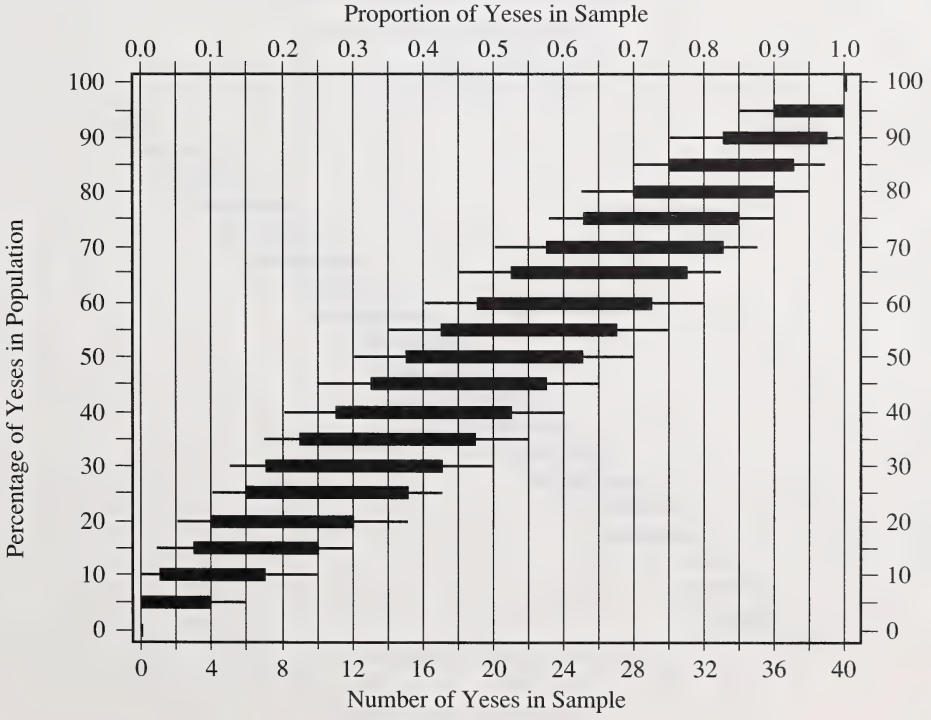
Areas under the Standard Normal Curve

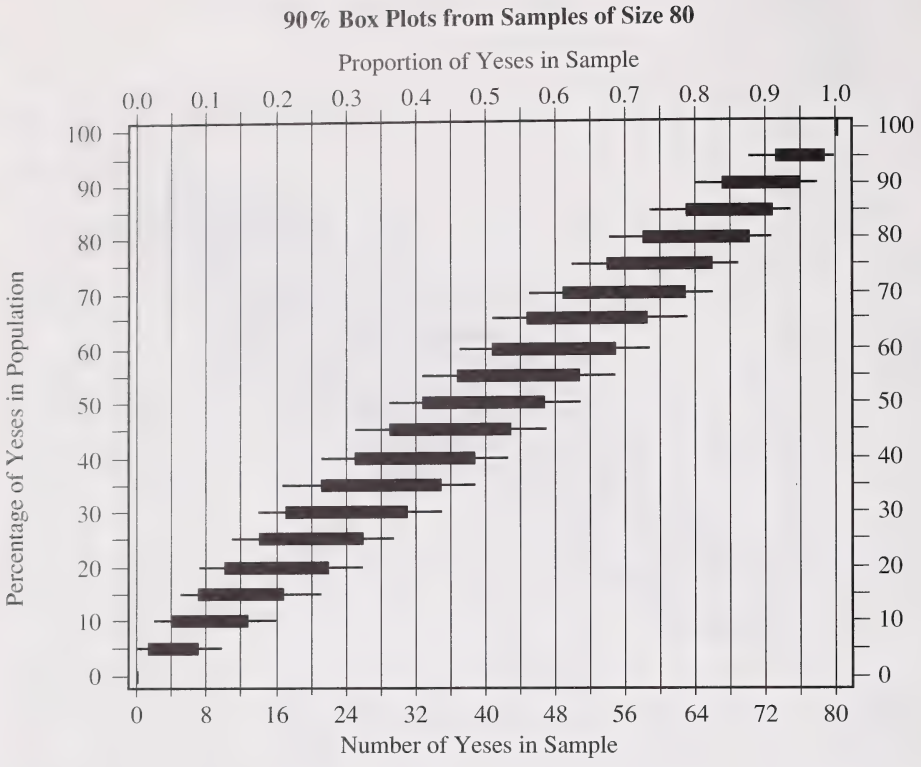
z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

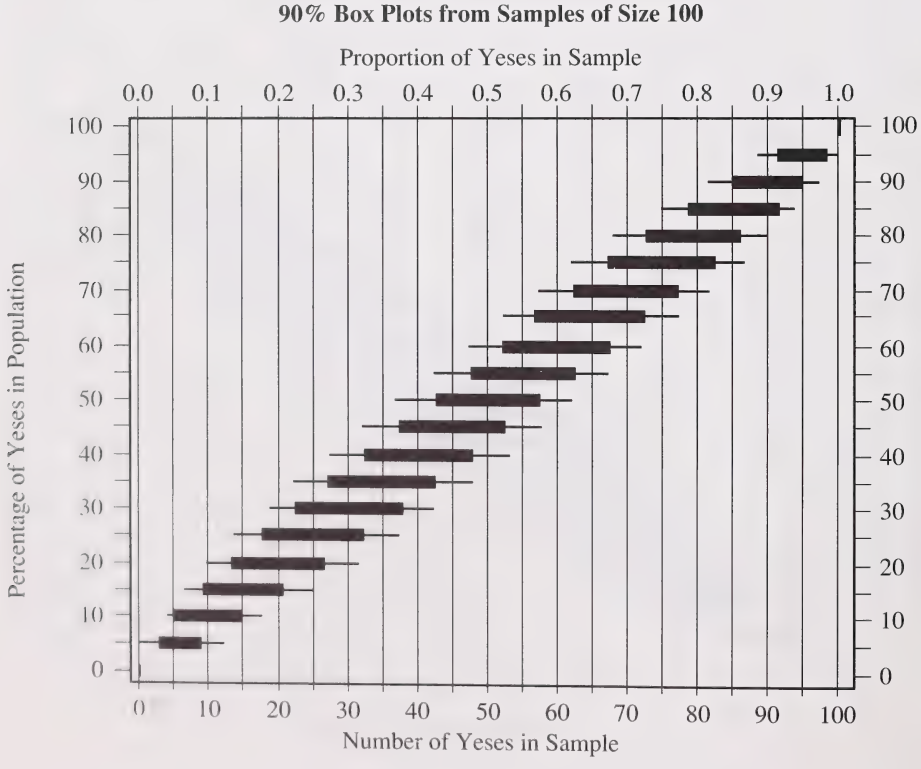
90% Box Plots from Samples of Size 20



90% Box Plots from Samples of Size 40




90% Box Plots from Samples of Size 80
Proportion of Yeses in Sample
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
100 90 80 70 60 50 40 30 20 10 0
Percentage of Yeses in Population
0 8 16 24 32 40 48 56 64 72 80
Number of Yeses in Sample


90% Box Plots from Samples of Size 100
Proportion of Yeses in Sample
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
100 90 80 70 60 50 40 30 20 10 0
Percentage of Yeses in Population
0 10 20 30 40 50 60 70 80 90 100
Number of Yeses in Sample

No marks will be given for work done on this page.

No marks will be given for work done on this page.

No marks will be given for work done on this page.

No marks will be given for work done on this page.

No marks will be given for work done on this page.

No marks will be given for work done on this page.

Mathematics 30

January 1993

Name

Apply Label With Student's Name

Mathematics 30

Name: (Last Name) (First Name) Date of Birth: Y M D Sex:

Permanent Mailing Address: (Apt./Street/Ave./P.O. Box) (Village/Town/City) (Postal Code)

School Code: School: Signature:

For Department Use Only

M1

M2

M3

M4

No Name

Apply Label Without Student's Name

Mathematics 30

MATHEMATICS 30 - 931

MULTIPLE-CHOICE KEY

1.	C	**22.**	D
2.	B	**23.**	D
3.	D	**24**	B
4.	C	**25.**	A
5.	B	**26.**	A
6.	D	**27.**	D
7.	A	**28.**	A
8.	B	**29.**	C
9.	B	**30.**	B
10.	D	**31.**	C
11.	C	**32.**	A
12.	C	**33.**	D
13.	A	**34.**	A
14.	B	**35.**	B
15.	A	**36.**	D
16.	C	**37.**	D
17.	A	**38.**	D
18.	C	**39.**	A
19.	B	**40.**	B
20.	B	**41.**	C
21.	C	**42.**	D

NUMERICAL-RESPONSE KEY

1.	5
2.	2.3
3.	95
4.	10.6
5.	18.1
6.	648
7.	22.0

SAMPLE ANSWERS TO THE WRITTEN-RESPONSE SECTION

Note: the responses that follow represent **ONE** approach to each of the problems. During the diploma examination marking session, provision is made for considering the various approaches students may have used.

1. The current population of the town of Winterlog is 3600. According to studies conducted over the last few years, Winterlog's population is growing at the rate of 1.6% per annum. Assuming that the population will continue to grow at this rate, determine how many **complete** years it will take Winterlog to reach a population of 4800.

3600	3600(1.016)	$3600(1.016)^2$ ...
NOW	Year 1	Year 2

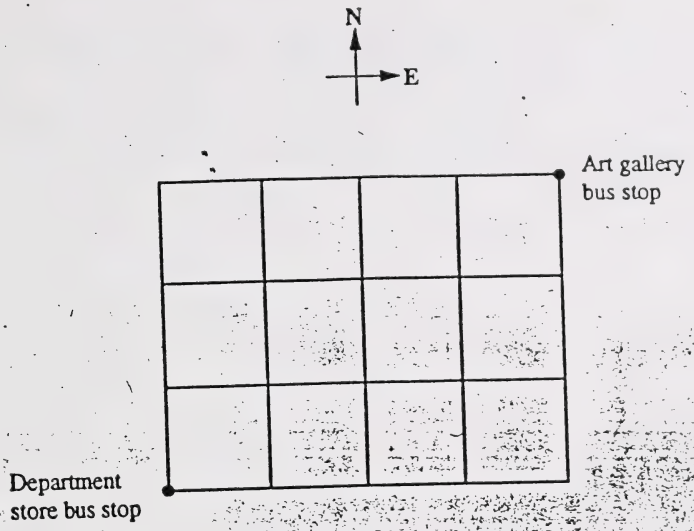
The sequence is geometric.

$r = 1.016$
$t_1 = 3600$
$t_n = 4800$

$$4800 = 3600(1.016)^{n-1}$$
$$\frac{4}{3} = (1.016)^{n-1}$$
$$\log\left(\frac{4}{3}\right) = (n-1)\log(1.016)$$
$$\frac{\log\left(\frac{4}{3}\right)}{\log(1.016)} + 1 = n$$
$$19.12 = n$$

It will take 19 years for Winterlog to reach a population of at least 4800, since $t_1 = 3600$ represents the current population of the town of Winterlog and not the population in one years time.

2. The head of city planning and development is designing a bus route from a department store to an art gallery. The art gallery bus stop will be three blocks north and four blocks east of the department store bus stop. The bus will travel only north or east.



Determine the total number of possible routes from the department store bus stop to the art gallery bus stop. Show how you solved this problem.

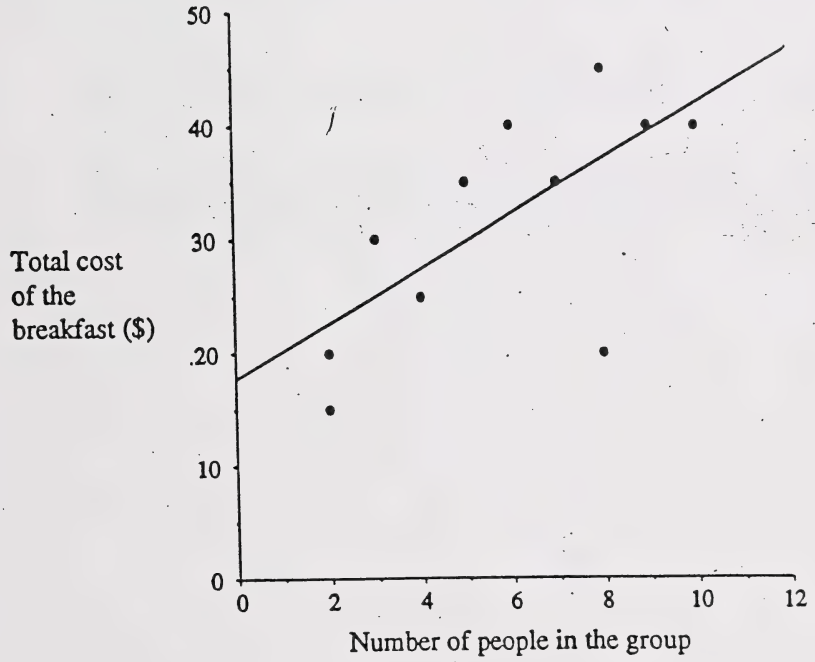
To get from the department store bus stop to the art gallery bus stop one must travel 3 blocks north and 4 blocks east in some order.

This is equivalent to the number of ways we can arrange:
N, N, N, E, E, E, E

Therefore the number of routes $= \frac{7!}{3!\,4!} = 35$ routes

3. Chewy Bits Cafe offers a variety of breakfast specials. Groups of people come to the cafe for these specials. The chart shows the number of people in 11 groups and the total cost of their breakfasts. The scatter plot and line of best fit are also shown.

Number of people in a group	2	2	3	4	5	6	7	8	8	9	10
Total cost of breakfast ($)	15	20	30	25	35	40	35	45	20	40	40



a. Use the line of best fit to predict the total cost of breakfast for a group of 8 people.

Approximately $38.00.

b. Use the line of best fit to determine how many people could eat breakfast for a total cost of $30.

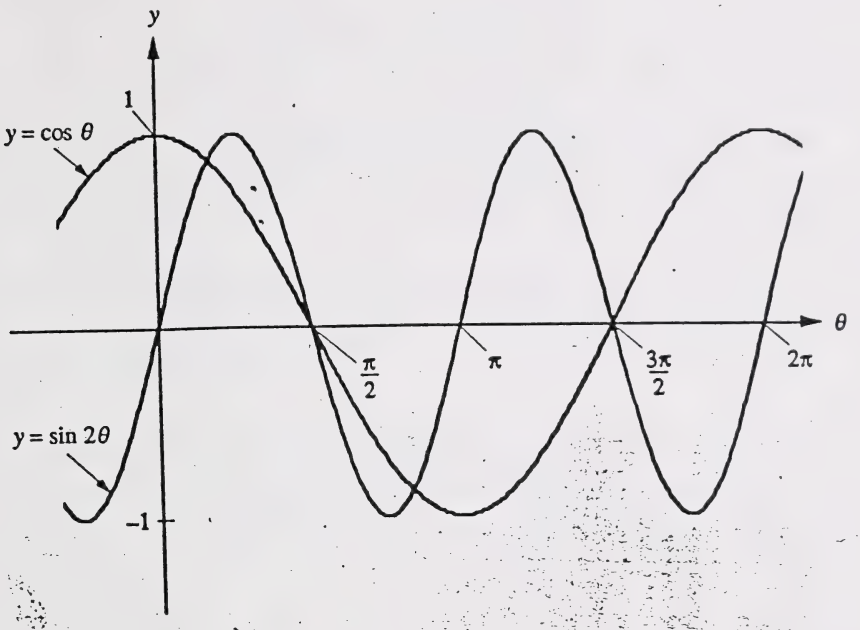
5 people

c. The line of best fit can be used to make predictions. Use the line of best fit to explain why the actual cost of a breakfast can differ significantly from the predicted cost.

The line of best fit is constructed using the entire set of data. It reflects all the data in the set rather than an individual value.

The actual cost of 3 people to eat breakfast was $30, whereas the predicted cost is approximately $25. The higher cost could be because someone ordered a more expensive breakfast.

4. Mike's assignment is to find the points of intersection of the graphs of $y = \sin 2\theta$ and $y = \cos \theta$, if $0 \leq \theta < 2\pi$. He uses his computer to graph $y = \sin 2\theta$ and $y = \cos \theta$. His computer display is



a. Looking at this computer display, Mike notices that one of the points of intersection is $\left(\frac{\pi}{2}, 0\right)$. He knows that it is impossible to find the exact values of all the points of intersection where $0 \leq \theta < 2\pi$ using the graphs displayed, so he decides to find the points of intersection algebraically. He also knows the trigonometric identity $\sin 2\theta = 2 \sin \theta \cos \theta$. Mike's first step in finding the points of intersection of the graphs is as follows:

> since $y = \sin 2\theta$ and $y = \cos \theta$ the θ values of the points of intersection are found when $\sin 2\theta = \cos \theta$.

Complete Mike's solution.

$$2\sin \theta \cos \theta = \cos \theta$$
$$2\sin \theta \cos \theta - \cos \theta = 0$$
$$\cos \theta (2\sin \theta - 1) = 0$$

$$\cos\theta = 0 \quad \text{or} \quad \sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or} \quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

points of intersection $\left(\frac{\pi}{2}, 0\right)\left(\frac{3\pi}{2}, 0\right)\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)\left(\frac{5\pi}{6}, \frac{-\sqrt{3}}{2}\right)$

b. How are the points of intersection of the graphs $y = 3\sin 2\theta$ and $y = 3\cos\theta$ related to the points of intersection of the graphs $y = \sin 2\theta$ and $y = \cos\theta$?

The points of intersection of the graphs $y = 3\sin 2\theta$ and $y = 3\cos\theta$ are in the form $(\theta, 3y)$ because the amplitude of both graphs changed from 1 to 3. So the θ values will be the same and the y values will be multiplied by 3.